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The Dirac-Ramond operator on loops in flat space. (English summary)

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In the 1980's, Witten formally applied the equivariant Atiyah-Singer index theorem to the formal Dirac operator on the loop space LM of a compact spin manifold M, with the natural S^1 action on LM as the rotation. In this way, he got the elliptic genera introduced by Landweber, Stong and Ochanine. Note that the elliptic genera motivated Landweber and Stong to define a new cohomology theory, the so-called "elliptic cohomology"; see [*Elliptic curves and modular forms in algebraic topology (Princeton, NJ, 1986)*, Lecture Notes in Math., 1326, Springer, Berlin, 1988; MR0970278] for the early history of the subject and [P. Landweber and K. Liu, editors, *Elliptic genera and elliptic cohomology: a reprint collection*, World Sci. Publishing, 2003; per revr.] for recent progress.

Since then, the rigorous construction of the Dirac operator on LM has remained open, and a solution of this problem in full generality would have a considerable impact in geometry and topology. One of the natural approaches is to use probability, such as Brownian motion in the probabilistic proof of the Atiyah-Singer index theorem [J.-M. Bismut, J. Funct. Anal. **57** (1984), no. 1, 56–99; MR0744920; R. Léandre, Probab. Theory Related Fields **80** (1988), no. 1, 119–137; MR0970474]. Léandre proposed a construction of the Dirac operator on LM when M is a compact homogenous space G/H [J. Math. Phys. **42** (2001), no. 3, 1364–1383; MR1814694].

In the present paper, the authors propose a construction of the Dirac operator on $L\mathbb{R}^d$ with the canonical flat metric on $\mathbb{R}^d$, and they compute its equivariant L^2-index, which is closely related to Dedekind's η-function. Instead of using probability, they use infinite tensor product representations of canonical anticommutation relation algebra.

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Note: This list reflects references listed in the original paper as accurately as possible with no attempt to correct errors.